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## LOCATION OF HEALTH CARE FACILITIES

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## **SUMMARY**

The location set covering model, maximal covering model and  $P$ -median model are reviewed. These models form the heart of the models used in location planning in health care. The health care and related location literature is then classified into one of three broad areas: accessibility models, adaptability models and availability models. Each class is reviewed and selected formulations are presented. A novel application of the set covering model to the analysis of cytological samples is then discussed. The chapter concludes with directions for future work.

## **KEY WORDS**

Facility location, Covering, Scenario planning

### 3.1 INTRODUCTION

The location of facilities is critical in both industry and in health care. In industry, poorly located facilities or the use of too many or too few facilities will result in increased expenses and/or degraded customer service. If too many facilities are deployed, capital costs and inventory carrying costs are likely to exceed the desirable value. If too few facilities are used, customer service can be severely degraded. Even if the correct number of facilities is used, poorly sited facilities will result in unnecessarily poor customer service.

In health care, the implications of poor location decisions extend well beyond cost and customer service considerations. If too few facilities are utilized and/or if they are not located well, increases in mortality (death) and morbidity (disease) can result. Thus, facility location modeling takes on an even greater importance when applied to the siting of health care facilities.

This chapter begins with a review of three basic facility location models from which most other models are derived: the set covering model, the maximal covering model, and the  $P$ -median model. Next, we discuss three major focal points of the location literature as it applies to health care facilities: accessibility, adaptability and availability. In the course of doing so, we review selected models and applications that have appeared in the literature. Our purpose is not to provide a comprehensive survey; rather our goal is to give the reader a feel for the models that have been proposed and the problems to which they have been applied. The reader interested in a more general introduction to facility location modeling should consult [1-4]. More recently Marianov and ReVelle [5] reviewed emergency siting models, Current, Daskin and Schilling [6] summarized general location models, Marianov and Serra [7] discussed the application of facility location models to problems in the public sector and Berman and Krass [8] summarized the state of the art in modeling problems with uncertainty and congestion, two issues we will return to below. We conclude the chapter by discussing an emerging health care application of facility location models that has nothing to do with the location of new physical facilities. We see such applications and adaptations of existing models as an important area for future research.

### 3.2 BASIC LOCATION MODELS

In this section we review three classic facility location models that form the basis for almost all of the facility location models that are used in health care applications. These are the set covering model, the maximal covering model, and the  $P$ -median model.

All three models are in the class of discrete facility location models, as opposed to the class of continuous location models. Discrete location models assume that demands can be aggregated to a finite number of discrete points. Thus, we might represent a city by several hundred or even several thousand points or nodes (e.g., census tracts or even census blocks). Similarly, discrete location models assume that there is a finite set of candidate locations or nodes at which facilities can be sited. Continuous location models assume that demands are distributed continuously across a region much the way peanut butter might be spread on a piece of bread. These models do not necessarily assume that demands are uniformly distributed, though this is a common assumption. Likewise, facilities can generally be located anywhere in the region in continuous location models. Throughout this chapter we restrict our attention to discrete location models since they have been used far more extensively in health care location problems.

At the heart of the set covering and maximal covering models is the notion of coverage. Demands at a node are generally said to be covered by a facility located at some other node if the distance between the two nodes is less than or equal to some exogenously specified coverage distance. Typically, the coverage distance is the same for all demand nodes, though additional restrictions on the set of candidate locations that can cover any particular demand node may be added. Such additional restrictions might reflect the ease of travel between population centers and a candidate site for a local clinic. For example, significant elevation changes might be penalized relative to flat terrain [9, 10]. Whether or not additional restrictions are placed on the cover sets, the mathematics is basically the same.

We define an indicator variable as follows:

$$a_{ij} = \begin{cases} 1 & \text{if demand node } i \text{ can be covered by a facility at candidate site } j \\ 0 & \text{if not} \end{cases}$$

The set covering model [11] attempts to minimize the cost of the facilities that are selected so that all demand nodes are covered. To formulate this model, we need the following additional sets and inputs.

$I$  = set of demand nodes

$J$  = set of candidate facility sites

$f_j$  = fixed cost of locating a facility at candidate site  $j$

In addition, we need the following decision variable.

$$X_j = \begin{cases} 1 & \text{if we locate at candidate site } j \\ 0 & \text{if not} \end{cases}$$

With this notation, we write the set covering problem as follows:

$$\text{Minimize} \quad \sum_{j \in J} f_j X_j \quad (1)$$

$$\text{Subject to} \quad \sum_{j \in J} a_{ij} X_j \geq 1 \quad \forall i \in I \quad (2)$$

$$X_j \in \{0,1\} \quad \forall j \in J \quad (3)$$

The objective function (1) minimizes the total cost of all selected facilities. Constraint (2) stipulates that each demand node must be covered by at least one of the selected facilities. The left hand side of (2) represents the total number of selected facilities that can cover demand node  $i$ . Finally, constraints (3) are standard integrality conditions.

In location problems, we are often interested in minimizing the number of facilities that are located, and not the cost of locating them. Such a situation might arise when the fixed facility costs are approximately equal and the dominant costs are operating costs that depend on the number of located facilities. In that case, the objective function becomes:

$$\text{Minimize} \quad \sum_{j \in J} X_j \quad (4)$$

To distinguish between these two model variants, we will refer to the problem with (1) as the objective function as the set covering problem or model; when (4) is used, we will call the problem the *location* set covering problem. A number of row and column reduction rules can be applied to the location set covering problem to reduce the size of the problem. Daskin [4] discussed and illustrated these rules.

In practice, at least two major problems occur with the set covering model. First, if (1) is used as the objective function, the cost of covering *all* demands is often prohibitive. If (4) is used as the objective function, the number of facilities required to cover all demands is often too large. Second, the model fails to distinguish between demand nodes that generate a lot of demand per unit time and those that generate relatively little demand.

Clearly, if we cannot cover all demands because the cost of doing so is prohibitive, we would prefer to cover those demand nodes that generate a lot of demand rather than those that generate relatively little demand. These two concerns motivated Church and ReVelle [12] to formulate the maximal covering problem. This model requires the following two additional inputs

$h_i$  = demand at node  $i$

$P$  = number of facilities to locate

as well as the following additional decision variable

$$Z_i = \begin{cases} 1 & \text{if demand node } i \text{ is covered} \\ 0 & \text{if not} \end{cases}$$

With this additional notation, the maximal covering location problem can be formulated as follows:

$$\text{Maximize} \quad \sum_{i \in I} h_i Z_i \quad (5)$$

$$\text{Subject to} \quad Z_i - \sum_{j \in J} a_{ij} X_j \leq 0 \quad \forall i \in I \quad (6)$$

$$\sum_{j \in J} X_j = P \quad (7)$$

$$X_j \in \{0,1\} \quad \forall j \in J \quad (8)$$

$$Z_i \in \{0,1\} \quad \forall i \in I \quad (9)$$

The objective function (5) maximizes the number of covered demands. Again, it is important to note that this model maximizes *demands* that are covered and not simply *nodes*. Constraint (6) states that demand node  $i$  cannot be counted as covered unless we locate at least one facility that is able to cover the demand node. Constraint (7) states that exactly  $P$  facilities are to be located and constraints (8) and (9) are standard integrality constraints.

A variety of heuristic and exact algorithms have been proposed for this model. In our experience, Lagrangian relaxation [13, 14] provides the most

effective means of solving the problem. When constraint (6) is relaxed, the problem decomposes into two separate problems: one for the coverage variables and one for the location variables. The subproblem for the coverage variables can be solved by inspection and the location variable subproblem requires only sorting. This approach can typically solve instances of the problem with hundreds of demand nodes and candidate sites to optimality in a few seconds or minutes on today’s computers even though the problem is technically NP-hard [15, 16]. Schilling, Jayaraman and Barkhi [17] reviewed the general class of location covering models.

The  $P$ -center model addresses the problem of needing too many facilities to cover all demands by relaxing the service standard (i.e., by increasing the coverage distance). This model finds the location of  $P$  facilities to minimize the coverage distance subject to a requirement that all demands are covered. Daskin [4] provided a traditional formulation of this problem. More recently, Elloumi, Labbé and Pochet [18] presented an innovative formulation of the problem that exhibits improved computational characteristics when compared to the traditional formulation.

The three models outlined so far – the location set covering model, the maximal covering location model, and the  $P$ -center model – treat service as binary: a demand node is either covered or not covered. While the notion of coverage is well established in health care applications, in many cases we are interested in the average distance that a client has to travel to receive service or the average distance that a provider must travel to reach his/her patients. To address such problems we turn to the  $P$ -median problem [19, 20], which minimizes the demand weighted total (or average) distance. To formulate this problem, we need the following additional input

$d_{ij}$  = distance from demand node  $i$  to candidate location  $j$

as well as the following new decision variable

$$Y_{ij} = \begin{cases} 1 & \text{if demands at node } i \text{ are assigned to a facility at candidate site } j \\ 0 & \text{if not} \end{cases}$$

With this notation, the  $P$ -median problem can be formulated as follows:

$$\text{Minimize} \quad \sum_{j \in J} \sum_{i \in I} h_i d_{ij} Y_{ij} \quad (10)$$

$$\text{Subject to} \quad \sum_{j \in J} Y_{ij} = 1 \quad \forall i \in I \quad (11)$$

$$Y_{ij} - X_j \leq 0 \quad \forall i \in I; \forall j \in J \quad (12)$$

$$\sum_{j \in J} X_j = P \quad (13)$$

$$X_j \in \{0,1\} \quad \forall j \in J \quad (14)$$

$$Y_{ij} \in \{0,1\} \quad \forall i \in I; \forall j \in J \quad (15)$$

The objective function (10) minimizes the demand weighted total distance. This is equivalent to minimizing the demand weighted average distance since the total demand is a constant. Constraint (11) states that each demand node must be assigned to exactly one facility site. Constraint (12) stipulates that demand nodes can only be assigned to open facility sites. Constraint (13) is identical to (7) above and states that we are to locate exactly  $P$  facilities. Constraints (14) and (15) are standard integrality constraints. Constraint (15) can be relaxed to a simple non-negativity constraint since each demand node will naturally be assigned to the closest open facility.

As in the case of the maximal covering problem, a variety of heuristic algorithms have been proposed for the  $P$ -median problem. The two best-known algorithms are the neighborhood search algorithm [21] and the exchange algorithm [22]. More recently, genetic algorithms [23], tabu search [24, 25] and a variable neighborhood search algorithm [26] have been proposed for this problem. Correa et al. [27] developed a genetic algorithm for a capacitated  $P$ -median problem in which each facility can serve a limited number of demands. They compared their algorithm to a tabu search algorithm and found that the genetic algorithm slightly outperformed the tabu search approach when the GA was accompanied by a heuristic hypermutation procedure. The latter simply performs an exchange algorithm on selected elements of the initial GA population and on population elements at a small number of randomly selected generations.

For moderate-sized problems, Lagrangian relaxation works quite well for the uncapacitated  $P$ -median problem. Constraint (11) is relaxed resulting in a set of subproblems for each candidate node that can easily be solved by inspection. Daskin [4] outlined the use of Lagrangian relaxation for both the  $P$ -median problem and the maximal covering model in detail. Daskin [28]



reported solution times for a Lagrangian relaxation algorithm for the  $P$ -median and vertex  $P$ -center problems with up to 900 nodes.

Some authors have transformed the maximal covering problem into a  $P$ -median formulation. This can be done by replacing the distance between demand node  $i$  and candidate site  $j$  by the following modified distance:

$$\hat{d}_{ij} = \begin{cases} 1 & \text{if } d_{ij} > D_c \\ 0 & \text{if not} \end{cases}$$

where  $D_c$  = the coverage distance. This has the effect of minimizing the total uncovered demand which is equivalent to maximizing the covered demand.

The uncapacitated fixed charge location (UFL) problem is a close cousin of the  $P$ -median problem. The UFL problem is derived from the  $P$ -median problem by eliminating constraint (13) and adding the objective function (1) to objective function (10) multiplied by a suitable constant to convert demand-miles into cost units. The problem then becomes that of determining the optimal number of facilities as well as their locations and the allocation of demands to those facilities to minimize the combined fixed facility location costs and the transport costs.

### 3.3 LOCATION MODELS IN HEALTH CARE

Having formulated three basic location models (the set covering model, the maximal covering model and the  $P$ -median model) and having qualitatively discussed two other classical models (the  $P$ -center problem and the uncapacitated fixed charge model) we now turn to applications and extensions of these models in health care. The health care location literature has tended to address three major topics, which we refer to as *accessibility*, *adaptability* and *availability*.

By accessibility we mean the ability of patients or clients to reach the health care facility or, in the case of emergency services, the ability of the health care providers to reach patients. Papers that deal with accessibility tend to ignore the needs of the system to evolve in response to changing conditions as well as short-term fluctuations in the availability of service providers as a result of their being busy serving other patients. Papers that focus on availability tend to be direct applications of one or more of the models above or are minor extensions of these models.

It is relatively easy and straightforward to site facilities based on a snapshot of the current or recent past conditions. Unfortunately, there is no guarantee that the future will replicate the past. Predicting future demand rates and operating conditions is exceptionally difficult. Thus, some recent applications and modeling efforts have focused on identifying solutions that can be implemented in the short term but that can adapt to changing future conditions relatively easily.

For some health care systems, and for emergency services in particular, some portion of the nominal capacity is likely to be unusable by new demands at any point in time as it is already in use by current demands. Thus, an ambulance may be busy responding to one emergency when another call for service within its district arises. To handle such situations, a significant literature has focused on designing systems to maximize some measure of the availability of the servers.

In short, accessibility models tend to take a snapshot of the system and plan for those conditions. As such, they are static models. Adaptability models often consider multiple future conditions and try to find good compromise solutions. As such, they tend to take a long-term view of the world. Availability models focus on the short-term balance between the ever-changing demand for services and the supply of those services.

### *3.3.1 Accessibility models and applications*

Accessibility models attempt to find facility locations that perform well with respect to static inputs. In particular, demand, cost and travel distance or travel time data are generally assumed to be fixed and non-random in this class of models. Thus, the models are often relatively straightforward extensions of the classic models outlined in section 1 above.

Indeed, federal legislation has encouraged the use of such models. The EMS (Emergency Medical Services) Act of 1973 stipulated that 95% of service requests had to be served within 30 minutes in a rural area and within 10 minutes in an urban area. This encouraged the use of models like the maximal covering model. Eaton et al. [29] used the maximal covering model to assist planners in Austin, TX in selecting permanent bases for their emergency medical service. The model was solved using the greedy adding and greedy adding and substitution algorithms. More recently, Adenso-Díaz and Rodríguez [30] also used the model to locate ambulances in Leon, Spain. They developed a tabu search algorithm to solve the problem.

Sinuany-Stern et al. [31] and Mehrez et al. [32] used two discrete models, the  $P$ -median model and a variant of the fixed charge location model in

which they constrained the travel time to any hospital and also invoke penalties for the assignment of demand to a hospital in excess of the hospital's capacity. These models were used, along with qualitative techniques, to generate alternative locations, which were then analyzed using the analytic hierarchy method. It is worth noting that the sites that were ultimately preferred tended to be those that were identified using one or more of the analytic methods, as opposed to those identified using qualitative techniques.

Jacobs, Silan and Clemson [33] used a capacitated  $P$ -median model to optimize collection, testing and distribution of blood products in Virginia and North Carolina. McAleer and Naqvi [34] also used a  $P$ -median model, in this case to relocate ambulances in Belfast, Ireland. Their problem was to locate four facilities to serve 54 demand nodes. The authors used a heuristic approach that decomposed the demand nodes into four sectors and ranked the possible single facility locations within each sector. This led to a number of acceptable solutions in each sector. All combinations of acceptable locations were then evaluated using all 54 demand nodes. While such a heuristic decomposition approach may make intuitive sense, it is not guaranteed to result in an optimal solution. Modern algorithms (e.g., Lagrangian relaxation embedded in branch and bound as implemented in SITATION [35]) can readily solve such problems to optimality on today's computers in seconds. Practitioners can also use such models to identify near optimal solutions, particularly when the number of facilities being located is small.

In hierarchical location modeling, a number of different services are simultaneously located. These might be, for example, local clinics, community health centers and regional hospitals. Lower level facilities (e.g., clinics) are generally assigned lower numbers (e.g., 1), while the highest level facilities (e.g., regional hospitals) are assigned the top number (e.g., 3). Another common application of hierarchical modeling is the location of basic life support vehicles (BLS or level 1 facilities) and advanced life support vehicles (ALS or level 2 facilities).

At least three factors need to be considered in hierarchical location problems [36]. The first is whether a level  $m$  facility can provide only level  $m$  service or whether or not it can also provide services at all lower levels (1, ...,  $m$ ). Clearly, an ALS vehicle can provide all levels of service that a BLS vehicle can provide. It is less clear that a regional hospital will be designed or staffed to provide all levels of support provided by a local clinic. For example, regional hospitals may not stock flu vaccines and, as such, may not be able to vaccinate individuals against the flu, while local clinics may be able to do so. A successively inclusive hierarchy is one in which a level  $m$

service can provide level  $m$  and all lower level services, while a successively exclusive hierarchy is one in which each level of service is provided by a unique facility. The second issue is, in a successively inclusive service, whether a level  $m$  facility can provide all  $m$  levels of service to *all* demand nodes, or a level  $m$  facility can provide all  $m$  levels of service only to demands at the node at which the facility is located and level  $m$  service only to other nodes. The former is referred to as a successively inclusive service hierarchy while the latter is termed a locally inclusive service hierarchy. A successively exclusive service hierarchy is one in which a level  $m$  facility provides only level  $m$  service to all nodes. Finally, there will generally be fewer high level facilities (e.g., regional hospitals) than low level facilities (e.g., local clinics). If high-level facilities can only be located at sites housing a lower level facility, the system is termed nested; otherwise it is not nested.

Finally, Price and Turcotte [37] used a center of gravity model to locate a blood donor clinic in Quebec. The model was used with a variety of inputs to identify a number of different locations from which a final choice was made. The center of gravity model minimizes the demand-weighted average distance between a facility that can be located anywhere in the plane and a discrete set of points. It is in the class of continuous location models (since the single facility location can be anywhere in the plane), which we are not explicitly reviewing and that have seen relatively little use in the health care location field. Nevertheless, Sinuany-Stern et al. [31] and Mehrez et al. [32] used two different continuous models in identifying candidate sites for a new hospital in the Negev. The first was the Weber model, which minimizes the demand weighted average Euclidean distance between a facility that can be anywhere in the plane and fixed demand locations, while the second was similar but used the square of the Weber objective function. (The reader interested in the Weber problem should consult the excellent review by Drezner et al.[38]).

### 3.3.2 *Adaptability models*

Location decisions must be robust with respect to uncertain future conditions, particularly for facilities such as hospitals that are difficult if not impossible to relocate as conditions change. A number of approaches have been developed to deal with future uncertainty. Scenario planning [39-41] is frequently used to handle future uncertainty. A number of future conditions are defined and plans are developed that do well in all (or most) scenarios.

In scenario planning, some decisions are made before the true scenario is revealed while others can be made after knowledge of the true scenario is gained. In location planning, the facility sites must generally be chosen

before we know which scenario will evolve; the assignment of demand nodes to sites can generally be done after we know which scenario will occur.

Designing a robust system often entails compromises. The “best” compromise plan may not be optimal under any particular scenario but will do well across all scenarios. The *regret* associated with a compromise solution and a scenario measures the difference between the performance measure using the compromise solution for that scenario and the performance measure when the optimal solution is used for that scenario.

Three performance measures are often used in scenario planning: optimizing the expected performance, minimizing the worst case performance, and minimizing the worst case regret. Minimizing the expected regret is identical to optimizing the expected performance.

In what follows, we formulate scenario-based extensions to the  $P$ -median problem. We define the following additional set and input

$S$  = set of scenarios

$q_s$  = probability that scenario  $s$  will occur

With this additional notation, the problem of minimizing the expected demand weighted total distance is formulated as follows, where we have added the subscript  $s$  to the demands and distances as well as the allocation variables:

$$\text{Min} \quad \sum_{s \in S} q_s \left\{ \sum_{j \in J} \sum_{i \in I} h_{is} d_{ijs} Y_{ijs} \right\} \quad (16)$$

$$\text{Subject to} \quad \sum_{j \in J} Y_{ijs} = 1 \quad \forall i \in I; \forall s \in S \quad (17)$$

$$Y_{ijs} - X_j \leq 0 \quad \forall i \in I; \forall j \in J; \forall s \in S \quad (18)$$

$$\sum_{j \in J} X_j = P \quad (19)$$

$$X_j \in \{0,1\} \quad \forall j \in J \quad (20)$$

$$Y_{ijs} \in \{0,1\} \quad \forall i \in I; \forall j \in J; \forall s \in S \quad (21)$$

The objective function (16) minimizes the expected demand weighted total distance over all scenarios. Constraint (17) states that each demand node is assigned to a facility in each scenario. Constraint (18) stipulates that these assignments can only be made to open facilities. Constraints (19) and (20) are identical to (13) and (14), respectively, and (21) is a standard integrality constraint.

To minimize the worst-case performance, the problem is restructured as follows:

$$\text{Min} \quad W \quad (22)$$

$$\text{Subject to} \quad W - \sum_{j \in J} \sum_{i \in I} h_{is} d_{ijs} Y_{ijs} \geq 0 \quad \forall s \in S \quad (23)$$

and (17) – (21)

where  $W$  is the maximum demand weighted total distance across all scenarios.

Finally, to minimize the maximum regret, we solve the following problem:

$$\text{Min} \quad V \quad (24)$$

$$\text{Subject to} \quad V - \left\{ \sum_{j \in J} \sum_{i \in I} h_{is} d_{ijs} Y_{ijs} - V_s^* \right\} \geq 0 \quad \forall s \in S \quad (25)$$

and (17) – (21)

where  $V_s^*$  is the optimal objective function value (smallest demand weighted total distance) for scenario  $s$ .

Both the minimax model (22)-(23) and the minimax regret model (24)-(25) avoid the need for scenario probabilities, which can be difficult to estimate. However, these models suffer from the fact that an unlikely scenario can drive the entire solution. At the other extreme, the problem of minimizing the expected performance (or equivalently the expected regret) tends to undervalue scenarios in which the compromise solution performs poorly if

those scenarios are low probability events. To handle these problems, Daskin, Hesse and ReVelle [42] introduced an  $\alpha$ -reliable minimax regret model. The model minimizes the maximum regret over an endogenously determined subset of the scenarios whose total probability must be at least  $\alpha$ .

Carson and Batta [43] considered the problem of locating an ambulance on the campus of the State University of New York at Buffalo in response to changing daily conditions. This is a particular problem on a large university campus since the center of gravity of the population shifts from dormitories to classrooms and offices over the course of the day. They determined that modeling four different time periods would suffice. By relocating the ambulance for each period, they were able to reduce the predicted average response time by 30% from 3.38 minutes (with a single static location) to 2.28 minutes (with four periods of unequal duration). The actual decrease in travel time when the solution was implemented was closer to 6% with the difference attributed to the non-linear nature of travel times. This work should not technically be viewed as part of the scenario planning literature since the decisions for each time period are unlinked. However, the work does highlight the value of being able to modify ambulance locations in response to changing daily conditions. The work also emphasized the need for careful modeling of travel time relationships, particularly when the average time is likely to be small.

ReVelle, Schweitzer and Snyder [44] proposed a number of variants of a conditional covering model in which demands at a node that houses a facility must be covered by a facility located elsewhere. In such models, the original demand nodes must be covered and each facility located by the model must be covered by a different facility. The rationale for such models is that if an emergency occurs at node  $j$  (e.g., an earthquake), then any emergency services at that location must be assumed to be damaged or unavailable for service at that node. Therefore, the node must be covered by some other facility.

In many important cases, the actual number of facilities that can be constructed in the long term is uncertain when the planning begins. Then, it is often important to be able to locate a known number of facilities now, accounting for the possibility that additional facilities could be built in future years. Current, Ratick and ReVelle [45] addressed this uncertainty with two models. In each model, the first stage decision entails locating  $P_0$  facilities now and  $P_s$  facilities in future state  $s$ , which occurs with probability  $\pi_s$ . The objective of the first problem is to minimize the expected opportunity loss (or regret) while the second problem minimizes the maximum regret. They illustrated the results using a small problem with 20 nodes, of which 10 were

candidate facilities, and 4 future states allowing for 0, 1, 2, or 3 additional facilities to be constructed. The models were solved using a standard LP/IP solver on a personal computer.

### 3.3.3 Models of Facility Availability

Adaptability reflects long-term uncertainty about the conditions under which a system will operate. Availability, in contrast, addresses very short-term changes in the condition of the system that result from facilities being busy. Such models are most applicable to emergency service systems (ambulances) in which a vehicle may be busy serving one demand at the time it is needed to respond to another emergency.

Deterministic models One simple, but somewhat crude, way of dealing with vehicle busy periods is to find solutions that cover demand nodes multiple times. The Hierarchical Objective Set Covering (HOSC) model [46] first minimizes the number of facilities needed to cover all demand nodes. Then, from among all the alternate optima to this problem – and there often are multiple alternate optima – the model selects the solution that maximizes the system-wide multiple coverage. The multiple coverage of a node is given by the total number of times the node is covered in addition to the one time needed to satisfy the set covering requirement. The system-wide multiple coverage is the sum of the nodal multiple coverage over all nodes. In essence, the model introduces an explicit surplus variable into constraint (2) and maximizes the sum of the surplus variables as a secondary objective to objective (4).

Benedict [47] modified the HOSC model to account for node demands and termed this **excess** coverage. To do so, he weighted the surplus variable by the node's demand. Eaton et al. [48] independently formulated and solved a similar model for locating ambulances in Santo Domingo. Hogan and ReVelle [49] considered a similar model that they termed **backup** coverage in which only a single additional cover of each node was counted and the additional cover of the node was weighted by the demand at the node.

Benedict also modified the maximal covering model to account for excess coverage. In this model the primary objective is to maximize the covered demand while the secondary objective is to maximize the excess coverage in the system. Benedict's third model was termed the hierarchical objective excess coverage model. In this model, the primary objective is to maximize excess coverage within T time units using the minimum number needed to cover all demand within T; the secondary objective is to maximize the demand that is covered within S, where S is less than T. Daskin, Hogan and



ReVelle [50] reviewed a variety of models of multiple, excess and backup coverage as well as the expected covering model discussed below.

Gendreau, Laporte and Semet [51] considered the problem of maximizing the number of demands that are covered by (at least) two ambulances in a distance  $r_1 < r_2$  while ensuring that each demand is covered within  $r_2$  and that at least  $\alpha\%$  of the demand is covered within  $r_1$ . A total of  $P$  ambulances are to be located. Like other multiple coverage models, this formulation is designed to increase the likelihood of there being an available ambulance within the coverage distance of a demand. Gendreau, Laporte and Semet solved the problem using tabu search for problem instances with up to 400 demand nodes and 70 candidate sites and 45 facilities.

Pirkul and Schilling [52] developed a model that minimizes the sum of the fixed facility costs, the costs of primary service and the costs of secondary service. Each demand node must be assigned to both a primary and a secondary facility. They developed a Lagrangian heuristic for solving the problem. The algorithm was embedded in a branch and bound algorithm to ensure optimality. They applied the algorithm to test problems ranging in size from 100 demand nodes and 10 candidate sites to 300 demand nodes and 30 candidate locations. They also tested the algorithm on a fire station location problem with 30 candidate sites and 625 demand nodes. By varying the weight on the fixed cost term of the objective function, the tradeoff between the number of facilities located and the average (primary and secondary) distance was identified for this larger problem.

Narasimhan, Pirkul and Schilling [53] considered the problem of locating a fixed number of facilities to maximize the amount of covered demand across a number of different levels of coverage, subject to a constraint that the total demand assigned to a facility across all levels of coverage cannot exceed a given value (the facility capacity). The model converts the maximal covering model into a  $P$ -median model and then introduces multiple levels of coverage and facility capacities. They argued that this “service level” can represent the order in which the facility providing service is called for service at a node. This is somewhat problematic since the order in which a facility at node  $j$  is called upon to respond to demands at node  $i$  depends on the location of other facilities, which is determined endogenously. Specifying this order exogenously seems extraordinarily difficult. They used a Lagrangian approach to solve the problem heuristically relaxing the assignment constraints. The authors solved the problem with up to 200 demand nodes, 30 candidate sites, 5 levels of service and 15 facilities being sited. Optimality gaps tended to be small, though for some (smaller) problems the maximum gap was 3 percent.

Probabilistic models The models discussed above take a deterministic approach to increasing the likelihood that a demand will be covered by an available vehicle or served adequately. Two different probabilistic approaches have been developed. The first approach is based on queuing theory while the second is based on Bernoulli trials.

Fitzsimmons [54] approximated the number of busy ambulances using an  $M/G/\infty$  queuing model. The average service time in his model depends on the number of busy vehicles, which, in turn, depends on the average service time. Thus, the two quantities are jointly estimated using an iterative sampling procedure. This is embedded in a search routine for finding improved ambulance locations. Eaton [55] provided an introduction to the use of this model in siting ambulances. While Fitzsimmons' approach can readily be embedded in a heuristic facility location model, it does not fully account for spatial differences in the probability of a vehicle being busy.

To address this shortcoming, Larson [56] developed a hypercube queuing model that accounts for spatially distributed service systems. The hypercube model is essentially an  $M/M/N$  queuing model with distinguishable servers. A binary string whose length is equal to the number of servers represents each state of the queuing system. For a system with  $n$  servers (ambulances) the model requires the solution of  $2^n$  simultaneous linear equations. Larson [57] proposed an approximation to the exact hypercube that entails solving  $n$  non-linear equations. Because of the difficulty in solving these models with *known* locations, they have tended not to be used in optimization modeling. Jarvis [58], however, embedded an approximation to the hypercube model in a heuristic search algorithm. Brandeau and Larson [59] used the hypercube model to locate ambulances in Boston.

An alternate, though less exact, approach involves representing the probability that a vehicle at any site  $j$  will be available as the outcome of a Bernoulli trial with probability of success (available) of  $q$ . Then, assuming that the probability  $q$  is the same throughout the system, the probability that all  $k$  vehicles that can cover a demand node  $i$  are busy is  $q^k$ . The probability that at least one of these  $k$  vehicles is available is  $1 - q^k$  and the incremental probability of at least one being available given that  $k$  vehicles can cover the demand node rather than just  $k-1$  vehicles is  $(1 - q^k) - (1 - q^{k-1}) = q^{k-1} - q^k = q^{k-1}(1 - q)$ .

This argument is at the heart of the maximum expected covering location model proposed by Daskin [60, 61]. To formulate this model, we define the following decision variable:

$$Y_{ik} = \begin{cases} 1 & \text{if demands at node } i \text{ are covered by } k \text{ or more vehicles} \\ 0 & \text{if not} \end{cases}$$

With this notation, the maximum expected covering model can be formulated as follows:

$$\text{Max} \quad \sum_{k=1}^P \sum_{i \in I} h_i q^{k-1} (1-q) Y_{ik} = (1-q) \sum_{k=1}^P \sum_{i \in I} h_i q^{k-1} Y_{ik} \quad (26)$$

$$\text{Subject to} \quad \sum_{k=1}^P Y_{ik} - \sum_{j \in J} a_{ij} X_j \leq 0 \quad \forall i \in I \quad (27)$$

$$\sum_{j \in J} X_j = P \quad (28)$$

$$X_j \in \{0, 1, \dots, P\} \quad \forall j \in J \quad (29)$$

$$Y_{ik} \in \{0, 1\} \quad \forall i \in I; k = 1, \dots, P \quad (30)$$

Under the independence assumption implicit in the Bernoulli trials model and the assumption that a single system-wide probability of a vehicle being busy ( $q$ ) can be estimated, the objective function (26) maximizes the expected demand covered by an available vehicle. Constraint (27) links the location variables to the coverage variables and states that a demand node cannot be counted as being covered  $k$  time unless there are at least  $k$  vehicles that can cover the node. Constraint (28) states that exactly  $P$  vehicles are to be located. Constraint (29) states that an integer number of vehicles must be located at any node, and constraint (30) states that the counting variables ( $Y_{ik}$ ) are binary. Note that constraint (29) does not restrict the number of vehicles at any location to be either 0 or 1. Daskin [61] proposed an exchange-based heuristic that approximates the solution for all values of  $q$ , the probability of a vehicle being busy.

Repende and Bernardo [62] extended the maximal expected covering location model to incorporate different time periods. The model allowed planners to reduce ambulance response time in Louisville, Kentucky, by 36%. They used simulation to validate the results of the time-variant expected covering model and to get better approximations of the actual expected coverage.

The maximum expected covering location model has two major limitations. Batta, Dolan and Krishnamurthy [63] showed that the independence assumption does not generally hold. They propose a number of ways of handling this including a formulation of an adjusted maximum expected covering location model that uses a correction term similar to that used by Larson [57] in developing the hypercube queuing model approximation. The second limitation of the maximum expected covering model has to do with the computation of the system-wide busy probability. Daskin [61] suggested computing system-wide busy probability as

$$q = \frac{\bar{t} \cdot \sum_i h_i}{24 \cdot P} \text{ where } \bar{t} = \text{average service time (in hours).}$$

ReVelle and Hogan [64, 65] extended the computation of the system-wide busy period to account for local conditions by approximating

$$q_i = \frac{\bar{t} \cdot \sum_{r \in M_i} h_r}{24 \sum_{j \in N_i} X_j} = \frac{F_i}{\sum_{j \in N_i} X_j} \quad (31)$$

where

$M_i$  = set of demand nodes that are within the coverage distance of node  $i$

$N_i$  = set of candidate sites that can cover demand node  $i$  and

$q_i$  = Probability that a vehicle located at  $i$  will be busy

With this local busy probability, ReVelle and Hogan [64] formulated the probabilistic set covering model as follows:

$$\text{Minimize } \sum_{j \in J} X_j \quad (32)$$

$$\text{Subject to } \sum_{j \in J} a_{ij} X_j = b_i \quad \forall i \in I \quad (33)$$

$$X_j \in \{0,1,\dots\} \quad \forall j \in J \quad (34)$$

In this model, node  $i$  must be covered  $b_i$  times, where  $b_i$  is the smallest value satisfying

$$1 - \left( \frac{F_i}{b_i} \right)^{b_i} \geq \alpha$$

and  $\alpha$  is the required probability of a node being covered by an available vehicle. Thus, this model is essentially a set covering model except that the right hand side of (33) is greater than 1 and we can locate more than one vehicle at a node. The model finds the minimum number of vehicles required to ensure that each demand node is covered by an available vehicle with probability  $\alpha$ , using the local busy probability estimates given by (31).

ReVelle and Hogan [64] defined the  $\alpha$ -reliable  $P$ -center problem and the maximum reliability location problem. The  $\alpha$ -reliable  $P$ -center problem finds the smallest coverage distance such that all demands are covered with probability  $\alpha$  by an available vehicle. This is solved by solving the problem above (32)-(34) for successively smaller values of the coverage distance until the objective function exceeds  $P$ . The maximum reliability location problem is to find the locations of  $P$  facilities such that the reliability  $\alpha$  is maximized. This can be solved by fixing a feasible value of  $\alpha$  and then solving the problem above. The value of  $\alpha$  is then increased until the required number of vehicles increases above  $P$ .

Similarly, ReVelle and Hogan [65] formulated the maximum availability location problem (MALP) as the problem of locating  $P$  vehicles to maximize the number of demands that are covered by an available vehicle with probability at least  $\alpha$ . Using the notation defined above, this model becomes:

$$\text{Maximize} \quad \sum_{i \in I} h_i Y_i b_i \quad (35)$$

$$\text{Subject to} \quad \sum_{k=1}^{b_i} Y_{ik} \leq \sum_{j \in J} a_{ij} X_j \quad \forall i \in I \quad (36)$$

$$Y_{ik} \leq Y_{i,k-1} \quad \forall i \in I; k = 2, \dots, b_i \quad (37)$$

$$\sum_{j \in J} X_j = P \quad (38)$$

$$X_j \in \{0, 1, \dots, P\} \quad \forall j \in J \quad (39)$$

$$Y_{ik} \in \{0, 1\} \quad \forall i \in I; k = 1, \dots, b_i \quad (40)$$

The objective function (35) maximizes the total demand that is covered by an available vehicle with probability at least  $\alpha$ . Constraint (36) links the coverage and location variables. Constraint (37) states that a node cannot be counted as being covered  $k$  times unless it is also counted as being covered  $k-1$  times. This constraint is not needed in the maximum expected covering problem since the decreasing value of the objective function coefficients for  $0 < q < l$  ensures that the coverage variables will enter the solution in this order. Constraint (38) states that  $P$  vehicles are to be located. Constraints (39) and (40) are integrality constraints. Again, we do not limit the number of vehicles located at a node to 0 or 1.

Ball and Lin [66] developed a model that is similar to the maximum availability location problem of ReVelle and Hogan [65], but do so from first principles. This helps identify the assumptions necessary for the development of the model. They then outlined a number of constraints that can be added to the formulation to tighten its linear programming relaxation, thereby facilitating the solution of the problem.

Goldberg et al. [67] developed a highly non-linear model that accounts for vehicle busy periods as a function of assignments. Assignments are for the  $k^{\text{th}}$  vehicle to respond to a demand in a region. The model was solved heuristically and was applied to the location of ambulances in Tucson, AZ. The model objectives include maximizing the number of calls responded to in 8 minutes (success rate), maximizing the worst node's success rate, and balancing workload. The approach was used primarily to evaluate a given set of sites though they did do some limited experimentation with an exchange algorithm.

Mandell [68] formulated a hierarchical ambulance location model in which demands are not covered unless either (1) a basic life support (BLS) unit can arrive at the scene within  $t^B$  and an advanced life support unit (ALS) can arrive within  $t^A$  with  $t^A > t^B$  or (2) an ALS unit can arrive within  $t^B$ . The model was formulated in terms of the probability that a demand is served

adequately given that there are  $r$  ALS units within  $t^A$ ,  $r'$  ALS units within  $t^B$  and  $s$  BLS units within  $t^B$ . Mandel used a two-dimensional Markov model (with states representing the number of ALS units within  $t^A$  of a demand node and the number of BLS units within  $t^B$  of the node) to estimate the required probabilities. The Markov model used demand-area specific arrival rates. The model was tested on a 55-node network. Computation times were under 1.5 seconds in all cases for the IP problem as formulated.

In the models described above, the primary objective was to account for vehicle busy periods. Another source of randomness arises from the location of the demands. Recognizing that demands occur over a region and not at discrete points, Aly and White [69] considered a probabilistic extension of the set covering model and of the  $P$ -median model. In both models the location of demands is uncertain, making the travel times random variables. Demand locations are uniformly distributed in rectangular regions. The distribution of travel time to a random point from a base with given coordinates is derived. From this the probability of being able to cover demands in a region from the base within a given time limit is derived. This results in the probabilistic set covering model – minimize the number of facilities need to ensure that each region is covered with probability  $\gamma$  – becoming a standard set covering model. Similarly, once we have the distribution of travel times, we can compute the expected travel time from a base at  $j$  with known coordinates to a point that is randomly distributed in some rectangular region  $i$ . This makes the probabilistic  $P$ -median problem – minimize the demand weighted expected travel time – a standard  $P$ -median problem as well. They concluded that the probabilistic formulation requires more facilities than does the deterministic formulation. Specifically, they stated, “In summary, using an aggregate point to represent a densely populated area may yield a less expensive siting cost. However, by ignoring the probabilistic element the actual service level will be much less than the one anticipated by the decision-maker.” (p. 1176)

Whether it arises from uncertain demand locations, vehicle busy periods, or changing and uncertain underlying conditions, stochasticity will degrade the performance of the system for a fixed set of resources.

### 3.4 ANOTHER APPLICATION OF LOCATION MODELS IN HEALTH CARE

The location set covering model – objective function (4) subject to (2) and (3) – has recently been used in a new health care application. Laporte et al. [70] reported on the use of this model to determine the minimum number of fields of view (FOV) to read a cytological sample (PAP test). A field of view is the area that a microscope can see without moving the slide being

analyzed. All areas of interest on a slide need to be examined (i.e., need to be in at least one FOV). At the same time, one would like to minimize the number of required FOVs so as to minimize the time needed to analyze each sample.

While the set covering model used by Laporte et al. is identical to that used in the location problems discussed above, there is an important difference. Typical location problems involve several hundred demand nodes and candidate locations. Solution time is not generally a problem in these instances because the problems are small and they do not have to be solved in real time. In the cytological example, the number of points to be covered can range from 2,500 to 55,000, approximately two orders of magnitude more than is typically found in a facility location example. Furthermore, the problems have to be solved very quickly as decisions about how to read a sample need to be made in real time. Furthermore, once appropriate FOVs have been identified, a routing problem needs to be solved to guide the microscope from one FOV to the next.

Laporte et al. [70] employed a series of heuristics to attack the problem. First, a mesh of FOVs was generated to cover all of the points of interest. Within each square, the smallest rectangle containing all of the points in the square was identified and up to four additional FOVs were generated, one located at each of the corners of this rectangle. A number of heuristics were then used to identify FOVs to include in the solution and others that could be excluded. Then a greedy heuristic proposed by Balas and Ho [71] was applied to solve the remaining problem. The routing heuristic was a straightforward adaptation of the strip heuristic proposed by Daganzo [72]. Solution times for the combined heuristic were typically under two minutes and thus were satisfactory for this application.

Brotcorne, Laporte and Semet [73] subsequently developed even faster heuristics for the tiling problem. It is worth noting that the best results in terms of a compromise between solution quality and execution time were generally not those that involve using the heuristic solution to the set covering model; instead, they used a variety of improvement heuristics.

### **3.5 SUMMARY AND DIRECTIONS FOR FUTURE WORK**

In this chapter we have presented the formulations of three location models that underlie most of the facility location models used in health care. The set covering model finds the minimum number (or cost) of facilities needed to cover all demands within a specified time or distance. The maximal covering location model relaxes the condition that all demands must be served within the covering standard and maximizes the number of covered



demands using a fixed number of facilities. Finally, the  $P$ -median model drops the notion of coverage and minimizes the demand-weighted total distance between demand nodes and the nearest facilities.

We identified three approaches to location modeling that have been used in health care applications. Accessibility models are typically straightforward extensions or applications of one of the basic location models. The goals of accessibility models are generally to maximize coverage or to minimize average distance. Adaptability models recognize that future conditions are difficult, if not impossible, to predict. These models attempt to find solutions that perform well across a range of future scenarios. Generally, a single set of locations must be identified for all scenarios, but the assignment of demands to facilities can be scenario-dependent. Typical objectives include optimizing the expected system performance, minimizing the worst-case performance, and minimizing the maximum regret. Regret measures the difference in the performance of the system for a given scenario between the compromise solution and the solution that would have been optimal for the specified scenario. Availability models attempt to account for the short-term unavailability of vehicles or facilities. Many such models have been applied to ambulance location problems. An ambulance might not be available when called upon for service because it is already serving another demand. A variety of deterministic, queuing-based and probabilistic availability models were reviewed.

We also outlined a health care application of the set covering model that results in problems that are approximately two orders of magnitude bigger than typical location problems and that has to be solved in real time. The application has to do with screening cytological samples and finding the minimum number of fields of view needed to read a sample.

In our view, the accessibility literature and the availability literature are quite mature, at least as applied to health care location problems. Considerably less work has been done on applying well-known concepts of scenario planning, or adaptability modeling, to health care problems. This seems to be a potentially fertile area for future work. Related to this is the area of reliability modeling. Reliability differs from adaptability in that adaptability (or robustness as it is sometimes termed) refers to the ability of a system to perform well in the face of uncertain future conditions. The uncertainty is typically in the input conditions including the costs and demands. Reliability, on the other hand, refers to the ability of a system to perform well when parts of the designed system fail [74]. Failures might result from capacity limitations or simply facility closures. Menezes, Berman and Krass [75] discussed reliability problems associated with Toronto hospitals. They noted that it is common for emergency rooms to be at capacity and to request

that the citywide system redirect emergencies to some other facility. Also, some hospitals were actually closed due to the SARS outbreak. Daskin and Snyder [76] presented two extensions of the  $P$ -median model designed to consider reliability, while Snyder [74] formulated and solved a variety of reliability extensions to location models. We believe that adaptability, robustness and reliability will become increasingly important in future applications in health care.

Finally, the application of location constructs to problems that do not involve locating any facilities seems to be an exciting area for future research and development. The use of location models in improving the efficiency of cytological diagnostic procedures outlined above is but one example of this line of research. Another application involves locating radioactive sources or seeds in the treatment of prostate cancer [77]. Applications of facility location-like models in the diagnosis and treatment of medical conditions is likely to be an important area of future work.

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