# A Hierarchical Objective Set Covering Model for Emergency Medical Service Vehicle Deployment[1]

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*The use of hierarchical and multiobjective programming in public decision making is reviewed. The conventional set covering (CSC) problem is formulated for locating emergency medical service (EMS) vehicles. Its computational and practical limitations are discussed. The desire to account for interdistrict responses leads to the formulation of a hierarchical objective set covering (HOSC) problem in which we find the minimum number of vehicles needed to cover all zones while simultaneously maximizing the extent of multiple coverage of zones. Several important properties of the HOSC problem are derived, including the fact that for certain values of the relative weights associated with the two objectives, no dominated zones are included in the solution. The CSC and HOSC formulations are applied to a 33-zone problem for Austin, Texas, and computational experiences are indicated.*

## 1. INTRODUCTION

Municipal service systems, such as police and fire departments and emergency medical services, must meet a variety of social, economic, and political objectives. Several authors have discussed the need to plan

[1] The work reported on here was performed while both authors were at the University of Texas at Austin.



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public facilities using a multiobjective framework.[5, 8, 10] To assist decision-makers, in identifying desirable modes of operating such services, analysts have used optimization techniques including multiobjective programming,[2] goal programming[14] and hierarchical programming.[19]

When a clear ranking of the objectives does not exist, multiobjective programming techniques are valuable tools in assisting decision-makers to identify tradeoffs between competing objectives. Multiobjective programming has been applied to problems in water resource planning[3, 4] as well as location problems.[2, 21] Goal programming techniques have been used to address such problems as school desegregation,[15] university staffing,[14] and the allocation of police patrol car units to districts.[20]

When a clear ranking or hierarchy of the objectives exists, hierarchical programming techniques are applicable. In this paper, we are concerned with two hierarchical objectives: the primary objective is to minimize the number of ambulances needed to satisfy a service requirement; the secondary objective is to maximize the extent of multiple coverage of zones. A hierarchical programming model is formulated and several model properties are proven. Computational experience with the model is also discussed.

Hierarchical programming has been used by several other authors in addressing location problems. PLANE AND HENDRICK[19] discuss the results of a study to locate fire stations. Their primary objective was to minimize the number of fire stations needed to reach all points in the specified time limits. Their secondary objective was to minimize the number of new fire stations that had to be built. KOLESAR AND WALKER[11] outline a hierarchical approach to the problem of relocating fire companies during busy periods. Their primary objective was to minimize the number of companies to be relocated while ensuring that a minimum level of fire protection was provided to all parts of the city. Their secondary objective was to minimize the total expected response time to alarms that occur while the companies are relocated.

## 2. PROBLEM STATEMENT

IN THIS PAPER we are concerned with locating emergency medical service (EMS) vehicles in a geographic region so as to cover all of the demands. More formally, we divide the region into $N$ zones and say that the demand in zone $i$ is covered by an EMS vehicle in zone $j$ if the expected response time for a vehicle in zone $j$ to a call in zone $i$ is less than or equal to some prespecified upper bound $T$. We require that all zones be covered by at least one vehicle. The response time may be defined in a variety of ways. The expected travel time between zone centroids is a frequently used proxy for response time. Use of this proxy necessitates several

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assumptions including the assumption that the expected dispatch delay is independent of the zone in which a call for service originates. We are not concerned here with the definition of response time that is used. We simply assume that the analyst knows the value of the expected response time, $d_{ij}$, for an ambulance in zone $j$ to respond to a demand in zone $i$ for all zone pairs $i$ and $j$. We note that we do not require $d_{ij} = d_{ji}$, although this will frequently be the case. Also, if institutional constraints prohibit an ambulance in zone $j$ from responding to demands in zone $i$ we may set $d_{ij} = \infty$.

Traditionally, this problem has been formulated as the set covering problem[1] in which one's objective is to minimize the number of vehicles required to cover all zones with at least one vehicle. Formally, the problem is stated as the following integer programming (IP) problem:

$$\text{minimize} \quad Z_1 = \sum_j X_j \tag{1}$$

subject to

$$\sum_j a_{ij} X_j \geq 1 \qquad \text{for all } i \tag{2}$$

$$X_j = 0, 1 \quad \text{for all } j \tag{3}$$

where

$$X_j = \begin{cases} 0 \text{ if an ambulance is not located in zone } j \\ 1 \text{ if an ambulance is located in zone } j \end{cases}$$

$$a_{ij} = \begin{cases} 0 \text{ if } d_{ij} > T \\ 1 \text{ if } d_{ij} \leq T \end{cases}$$

and

$$Z_1 = \text{the number of ambulances required.}$$

Similar set covering formulations have been used by others in the context of locating emergency service vehicles.[11, 19, 22, 23] We let $A(T)$ be the matrix of $a_{ij}$ terms for a maximum allowable response time $T$ and call $A(T)$ the coverage matrix. We omit the notation $T$ whenever possible. It should be clear that as the upper bound on the allowable response time, $T$, is reduced, the number of ambulances needed will increase or remain constant.

One approach to solving this problem is to replace the integer constraint (3) with a non-negativity constraint and to solve the resulting linear programming (LP) problem. The LP algorithm may terminate with

a. No feasible solution

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b. A noninteger value for the objective function (and at least one of the decision variables, $X_j$, also noninteger)
c. An integer value for the objective function and noninteger values for the decision variables, or
d. Both the objective function and the decision variables integer.

No feasible solution will be found only if there exists at least one zone $i$ such that $\sum_j a_{ij} = 0$, which implies that $a_{ii} = 0$. This can occur if the intrazonal response time $d_{ii}$ is very large. In such a case, a redefinition of the zone structure is needed to reduce the intrazonal response times. We assume that $\sum_j a_{ij} \geq 1$ for all zones $i$, or alternatively that there exists some $d_{ij} \leq T$ for all zones $i$. Generally, we will have $d_{ii} \leq T$ and $a_{ii} = 1$ for all zones $i$.

If the LP algorithm terminates in a noninteger solution (cases (b) and (c) above), the LP solution may be used as a lower bound on the value of the objective function or the IP problem. TOREGAS ET AL.[22] report success in obtaining integer solutions in such cases through the addition of the constraint

$$\sum_j X_j \geq \langle Z_{1L} \rangle + 1 \tag{4}$$

where

$$Z_{1L} = \text{value of the LP objective function}$$

$$\langle y \rangle = \text{largest integer less than or equal to } y.$$

If the optimal LP objective function is an integer, but the decision variables are noninteger, using the LP solution as a lower bound on the IP objective function does little to guide the search for integer solutions. This is illustrated by the coverage matrices shown in Figure 1. In both cases, one solution at which the LP algorithm may terminate is

$$X_1 = X_2 = X_3 = X_5 = ½; Z = 2.$$

In Case I, integer solutions using only two vehicles exist ($X_1 = X_5 = 1$ or $X_2 = X_3 = 1$). In Case II, no such integer solution exists; three ambulances are needed to cover all 6 zones. (One such solution is $X_1 = X_2 = X_3 = 1$.) Other examples of this problem are illustrated in Tables I and II below when the maximum allowable travel time is 9 or 14 minutes. In such situations, special set covering algorithms[1] or heuristics[6] may be needed.

In addition to computational difficulties associated with obtaining integer solutions for the IP set covering problem, the traditional formulation (1), (2), and (3) fails to account explicitly for interdistrict responses. In effect, the algorithm assumes that the vehicle that is assigned to cover some zone $i$ will always be available to respond to a call from that zone

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CASE I:

$$A^{I} = [a^{I}_{ij}] = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

CASE II:

$$A^{II} = [a^{II}_{ij}] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

**Fig. 1.** Example coverage matrices.

and will never be busy. In practice, however, this will not be the case. Frequently the most desirable ambulance to dispatch to a call in zone $i$ will be busy when a call from zone $i$ is received. In such a case an interdistrict response is needed.

Many stochastic models of EMS vehicle deployment explicitly account for interdistrict calls.[7, 12, 13] Deterministic optimization approaches like the set covering formulation, the median problem[9, 16] or the center problem[9, 16] do not model interdistrict responses. Recent stochastic extensions to the median problem[17, 18] also ignore interdistrict responses. Because of the occurrence of interdistrict dispatches, we would like the set of locations that is selected to provide as much multiple coverage as is possible. In that way, the system will have the capability to respond to calls expeditiously even if the most desirable ambulance to dispatch is busy at the time a call is received.

The traditional single objective set covering problem can be modified to incorporate two objectives:

a. Minimize the number of ambulances that are required to cover each of the $N$ zones in time $T$ and
b. Given the minimum number of ambulances, maximize the sum over all zones $i$ of the number of ambulances in addition to the one required by objective (a) above that can respond to calls in zone $i$ within time $T$.

Objective (a) is the traditional set covering objective. Objective (b) is

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designed to select from among the alternative optima for objective (a) the one that maximizes the amount of multiple coverage in the *system.* Note that objective (b) weights equally all ambulances that can respond to a call in zone *i*. From a practical point of view, one might like to use a decreasing set of weights for additional ambulances; that is, one might like to weight the second ambulance more than the third; the third more than the fourth; and so on. For computational reasons, we do not adopt this approach.

## 3. MODEL FORMULATION

FORMALLY, we modify the CSC problem as follows:

$$\text{minimize} \quad Z_2 = W \textstyle\sum_j X_j - \sum_i S_i \tag{5}$$

subject to

$$\textstyle\sum_j a_{ij} X_j - S_i \geq 1 \quad \text{for all } i \tag{6}$$

$$X_j = 0, 1 \quad \text{for all } j \tag{7a}$$

$$S_i \geq 0 \quad \text{for all } i \tag{7b}$$

where

$S_i$ = number of additional EMS units capable of responding to a call in zone $i$ in a time less than or equal to $T$

$W$ = some positive weight

and all other variables are as defined above. We call this the hierarchical objective set covering (HOSC) problem. Plane and Hendrick[19] used a similar formulation in their problem of locating fire stations.

## 4. SOLUTION CHARACTERISTICS

THE MODEL of Section 3 has several desirable properties. Many of these properties depend on the value of $W$, the weight associated with minimizing the number of vehicles deployed. We begin with

PROPOSITION 1. *For any value of W, inequality* (6) *will be satisfied by a strict equality for all zones i in the optimal solution.*

*Proof.* Suppose there exists some zone $i$ for which the left hand side of (6) exceeds 1. Then $S_i$ can be increased by the difference between the left hand side and 1. This will reduce the objective function by an equal amount. Hence, the original solution (in which the left hand side of (6) exceeded 1) could not be optimal.

One approach to solving the IP problem is to substitute a non-negativ-

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ity constraint for (7a) and to solve the resulting LP problem. The following proposition applies to this LP problem.

PROPOSITION 2. *If $W > N$, then $0 \le X_j^* \le 1$ for all $j$, where stars (*) indicate optimal values.*

*Proof.* (i) $X_j^* \ge 0$ for all $j$ by the non-negativity constraint.

(ii) let $X_j^* = 1 + \delta$, $\delta > 0$. Consider reducing $X_j^*$ to 1. This reduces $S_i$ by $\delta\, a_{ij}$, for all $i$. So $\sum S_i$ is reduced by

$$\delta \textstyle\sum_i a_{ij} \le N\delta.$$

Therefore, the value of the objective function is reduced by at least $\delta(W - N)$. Since $\delta > 0$ and $W > N$ by assumption, $Z_2$ is reduced by a positive amount. Therefore, $X_j = 1 + \delta$, $\delta > 0$ can not be optimal.

We say that zone $k$ is dominated by zone $j$ if all zones covered by zone $k$ are also covered by zone $j$ and zone $j$ covers at least one zone not covered by zone $k$. Formally, $j$ dominates $k$ if

$$a_{ij} \ge a_{ik}, \qquad \text{for all } i$$

and

$$a_{ij} > a_{ik}, \qquad \text{for at least one zone } i.$$

This definition leads to

PROPOSITION 3. *If $W > N$, the optimal solution to the HOSC problem will not include any dominated zones.*

*Proof.* Assume zone $j$ dominates zone $k$. Let $c_j = \sum_i a_{ij}$ = number of zones an ambulance in zone $j$ can cover. By definition $c_j \le N$ for all zones $j$ and by dominance, $c_k + 1 \le c_j \le N$.

*Case (i).* Assume $X_j = 1$, $X_k = \epsilon > 0$ in the optimal solution. By dominance, $X_j = 1$, $X_k = 0$ is feasible if $X_j = 1$ and $X_k = \epsilon$ is feasible. Let

$$Z_0 = \text{value of the objective function when } X_k = 0$$

$$Z_\epsilon = \text{value of the objective function when } X_k = \epsilon.$$

Then

$$\begin{aligned} Z_\epsilon &= Z_0 + W\epsilon - c_k\epsilon \\ &= Z_0 + (W - c_k)\epsilon \\ &> Z_0 + \epsilon. \end{aligned}$$

But since $Z_0$ is feasible, $Z_\epsilon$ or $X_k = \epsilon > 0$ can not be optimal. In other words, the dominated zone can not be in the optimal basis for the LP problem.

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*Case* (*ii*). Assume $X_j = \gamma < 1$, $X_k = \epsilon > 0$ in the optimal solution. Consider increasing $X_j$ by $\delta \le \min(\epsilon, 1 - \gamma)$ and decreasing $X_k$ by $\delta$. By dominance $X_j' = \gamma + \delta$ and $X_k' = \epsilon - \delta$ are feasible. Define $Z_\epsilon$ and $Z_{\epsilon-\delta}$ in the obvious manner in terms of the value of $X_k$ and $X_k'$. Then

$$\begin{aligned} Z_{\epsilon-\delta} &= Z_\epsilon - c_j\delta + c_k\delta \\ &= Z_\epsilon - \delta(c_j - c_k) \\ &\le Z_\epsilon - \delta. \end{aligned}$$

Since we may choose $\delta > 0$, the original solution ($X_j = \gamma < 1$, $X_k = \epsilon > 0$) can not be optimal. If $\epsilon \le 1 - \gamma$, we can set $\delta = \epsilon$ and obtain $X_k = 0$ thereby forcing $X_k$ from the basis. If $\epsilon > 1 - \gamma$, we can set $\delta = 1 - \gamma$ and force $X_j = 1$, $X_k = \epsilon - 1 + \gamma > 0$. But by case (i) above we can force $X_k$ to 0 in this case as well.

## 5. CHOICE OF $W$

As shown below in Section 6, the hierarchical objective set covering formulation leads to an all integer solution more often than does the conventional set covering formulation. This is in part due to the automatic exclusion of dominated zones in the hierarchical objective formulation; the solution to the LP analog of the CSC problem may include dominated zones.

While the hierarchical objective formulation excludes dominated zones from the solution, it does not necessarily result in a solution utilizing the minimum number of vehicles. This results directly from the fact that the objective function involves two conflicting objectives. This is illustrated by the example in Figure 2. It is clear that only two vehicles are required to cover the 11 zones in the example. One allocation of 2 vehicles that minimizes objective function $Z_2$ in (5) is $X_4 = X_5 = 1$ (Solution 1). The resulting objective function value is 22. However, the HOSC problem when constrained to integer solutions will terminate with $X_2 = X_3 = X_4 = 1$ (Solution 2) and an objective function value of 21. This difficulty may be alleviated by increasing the value of the weight $W$ associated with the number of vehicles. For example, if we had used $W = 14$ instead of $W = 12$, we would have $Z_2 = 26$ for $X_4 = X_5 = 1$ (Solution 1) and $Z_2 = 27$ for $X_2 = X_3 = X_4 = 1$ (Solution 2). The IP algorithm would terminate with Solution 1, which uses the minimum number of ambulances.

This leads to

PROPOSITION 4. *Let* $\{X_j^*, S_i^*\}$ *be the optimal solution to the HOSC problem* (5), (6), (7b) *and either* (7a) *or its non-negativity analog. Let* $\{X_j', S_i'\}$ *be any other feasible solution to the same form of the HOSC*

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$$A = [a_{ij}] = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$N = 11$ zones

$W = 12$

Solution 1: $X_4 = X_5 = 1$ all other $X_j = 0$; $Z_2 = 22$

Solution 2: $X_2 = X_3 = X_4 = 1$ all other $X_j = 0$; $Z_2 = 21$

Note 1: HOSC may not lead to the deployment of the minimum number of vehicles.

Note 2: When the LP analogies of the CSC and HOSC formulations were applied to this problem, the following results were obtained.

CSC: $X_1 = X_2 = 1$ all other $X_j = 0$; $Z_{1L} = 2$

HOSC: $W = 12$

$X_2 = X_3 = X_4 = X_5 = 0.5$ all other $X_j = 0$

$Z_{2L} = 20$

$W = 14$

$X_2 = X_3 = X_4 = X_7 = 0.5$ all other $X_j = 0$

$Z_{2L} = 24$

**Fig. 2.** Example to illustrate the influence of $W$ on the optimal solution.

*problem. If* $\sum S_i^* < W$, *then* $\sum X_j' > \sum X_j^* - 1$, *or equivalently* $\sum X_j' \geq \sum X_j^*$ *in the IP problem.*

*Proof.* Assume $\sum X_j' \leq \sum X_j^* - 1$ or equivalently

$$\sum X_j' - \sum X_j^* + 1 \leq 0.$$

Optimality of $\{X_j^*, S_i^*\}$ implies

$$W \sum X_j^* - \sum S_i^* \leq W \sum X_j' - \sum S_i'$$

or

$$\sum S_i' \leq W(\sum X_j' - \sum X_j^*) + \sum S_i^*$$
$$< W(\sum X_j' - \sum X_j^*) + W$$

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since $\sum S_i^* < W$ by assumption. So

$$\sum S_i' < W(\sum X_j' - \sum X_j^* + 1) \le 0$$

by the assumption that $\sum X_j' \le \sum X_j^* - 1$. But this states that $\sum S_i' < 0$ which contradicts the assumption that $\{X_j', S_i'\}$ is feasible since feasibility requires $S_i' \ge 0$ for all $i$ or $\sum S_i' \ge 0$. Therefore we have

$$\sum X_j' > \sum X_j^* - 1 \quad \text{if} \quad \sum S_i^* < W.$$

The implications of Proposition 4 warrant further discussion. The proposition says that if the solution obtained to the HOSC problem involves only integer valued decision variables and if $\sum S_i^* < W$, then the solution is also an optimal solution to the CSC problem. That is, the solution employs the minimum feasible number of vehicles. The proposition further implies that if $\sum X_j^*$ in the solution to the LP analog of the HOSC problem is noninteger and $\sum S_i^* < W$, then $\langle \sum X_j^* \rangle$ provides a lower bound on the IP solution to the CSC problem. This is true because feasibility in the CSC problem implies and is implied by feasibility in the HOSC problem.

Finally, if $\sum S_i^* \ge W$ in a solution to the HOSC problem, Proposition 4 suggests that $W$ be increased to $\hat{W}$ where $\sum S_i^* < \hat{W}$. In the solution to the revised problem with $\sum S_i^* < \hat{W}$, we are assured that $\sum \hat{S}_i^* < \hat{W}$ where $\hat{S}_i^*$ = optimal value of $S_i$ in the revised problem using $\sum S_i^* < \hat{W}$. Therefore, Proposition 4 will apply to the solution of the revised problem.

In short, while the solution to the HOSC problem may not employ the minimum number of vehicles, Proposition 4 allows us to make fairly strong statements about the minimum number when $\sum S_i^* < W$. The proposition also provides important guidance on appropriate settings for $W$ if one wishes to minimize the number of vehicles deployed. Also, Proposition 3 guarantees that if $W > N$, the solution to the HOSC problem will not include any dominated zones.

## 6. COMPUTATIONAL EXPERIENCE

THE HOSC formulation was applied to data for Austin, Texas. The city was divided into 34 zones corresponding to census tracts. After some preliminary analysis and as a result of discussions with the director of the Austin EMS service, one of the zones was deleted from further analyses. Few demands for EMS services were recorded in this zone, yet the HOSC and CSC formulations always located an ambulance in the zone because it was remote from the city center. The results discussed below are for the 33-zone problem obtained by eliminating this particular zone.

As discussed above, in Section 2, we use expected travel time as a proxy for response time. Figure 3 is the travel time matrix for the 33 zones. All intrazonal travel times were set equal to 1 minute.

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TO

| FROM | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 5 | 8 | 11 | 8 | 8 | 9 | 12 | 13 | 13 | 9 | 8 | 11 | 12 | 13 | 6 | 9 |
| 2 | | 1 | 4 | 7 | 4 | 4 | 6 | 9 | 11 | 10 | 7 | 7 | 9 | 10 | 11 | 7 | 7 |
| 3 | | | 1 | 6 | 5 | 5 | 6 | 7 | 9 | 9 | 7 | 11 | 12 | 12 | 10 | 8 | 8 |
| 4 | | | | 1 | 6 | 5 | 4 | 3 | 5 | 7 | 5 | 9 | 10 | 10 | 7 | 13 | 12 |
| 5 | | | | | 1 | 3 | 5 | 7 | 9 | 9 | 6 | 9 | 10 | 10 | 9 | 9 | 10 |
| 6 | | | | | | 1 | 3 | 6 | 8 | 7 | 4 | 6 | 8 | 8 | 8 | 10 | 10 |
| 7 | | | | | | | 1 | 4 | 6 | 5 | 2 | 5 | 7 | 7 | 6 | 11 | 12 |
| 8 | | | | | | | | 1 | 4 | 5 | 4 | 8 | 9 | 9 | 7 | 14 | 13 |
| 9 | | | | | | | | | 1 | 4 | 5 | 8 | 9 | 9 | 6 | 15 | 15 |
| 10 | | | | | | | | | | 1 | 4 | 7 | 7 | 6 | 3 | 15 | 15 |
| 11 | | | | | | | | | | | 1 | 4 | 5 | 5 | 5 | 11 | 13 |
| 12 | | | | | | | | | | | | 1 | 5 | 7 | 7 | 10 | 13 |
| 13 | | | | | | | | | | | | | 1 | 3 | 6 | 13 | 15 |
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| 31 | | | | | | | | | | | | | | | | | |
| 32 | | | | | | | | | | | | | | | | | |
| 33 | | | | | | | | | | | | | | | | | |

SYMMETRIC TRAVEL TIME

MATRIX USED

TO

| FROM | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 10 | 6 | 7 | 9 | 16 | 14 | 16 | 11 | 10 | 15 | 13 | 16 | 18 | 15 | 19 | 22 |
| 2 | 7 | 5 | 6 | 11 | 15 | 12 | 13 | 10 | 9 | 14 | 10 | 12 | 14 | 13 | 16 | 19 |
| 3 | 6 | 9 | 10 | 13 | 19 | 11 | 10 | 10 | 13 | 16 | 7 | 10 | 12 | 12 | 14 | 18 |
| 4 | 9 | 10 | 9 | 17 | 17 | 13 | 12 | 13 | 12 | 14 | 9 | 7 | 9 | 10 | 10 | 16 |
| 5 | 8 | 7 | 8 | 13 | 17 | 14 | 13 | 12 | 11 | 15 | 9 | 11 | 13 | 11 | 14 | 18 |
| 6 | 9 | 6 | 6 | 14 | 16 | 14 | 13 | 13 | 9 | 13 | 10 | 11 | 13 | 10 | 14 | 16 |
| 7 | 10 | 7 | 6 | 14 | 15 | 14 | 13 | 14 | 9 | 11 | 10 | 9 | 12 | 8 | 12 | 14 |
| 8 | 11 | 10 | 9 | 18 | 17 | 14 | 14 | 14 | 12 | 13 | 10 | 7 | 10 | 9 | 9 | 15 |
| 9 | 13 | 11 | 10 | 19 | 16 | 17 | 16 | 16 | 12 | 13 | 12 | 7 | 11 | 8 | 8 | 13 |
| 10 | 12 | 11 | 9 | 18 | 13 | 16 | 15 | 16 | 11 | 10 | 12 | 10 | 14 | 5 | 10 | 12 |
| 11 | 11 | 7 | 5 | 15 | 13 | 15 | 14 | 15 | 7 | 9 | 11 | 9 | 13 | 7 | 11 | 13 |
| 12 | 13 | 6 | 4 | 14 | 13 | 18 | 17 | 15 | 6 | 11 | 14 | 13 | 16 | 10 | 14 | 16 |
| 13 | 14 | 9 | 7 | 16 | 10 | 19 | 18 | 17 | 7 | 8 | 15 | 14 | 17 | 8 | 14 | 14 |
| 14 | 15 | 10 | 8 | 17 | 10 | 19 | 18 | 19 | 8 | 7 | 15 | 14 | 17 | 6 | 13 | 12 |
| 15 | 13 | 10 | 9 | 18 | 12 | 17 | 16 | 17 | 10 | 9 | 13 | 11 | 15 | 4 | 10 | 11 |
| 16 | 6 | 8 | 9 | 7 | 18 | 11 | 12 | 6 | 12 | 17 | 10 | 17 | 16 | 18 | 21 | 24 |
| 17 | 5 | 10 | 11 | 9 | 21 | 10 | 11 | 6 | 14 | 19 | 9 | 16 | 15 | 18 | 20 | 24 |
| 18 | 1 | 11 | 12 | 12 | 21 | 9 | 10 | 7 | 15 | 20 | 6 | 14 | 12 | 15 | 17 | 21 |
| 19 | | 1 | 4 | 11 | 14 | 16 | 17 | 12 | 7 | 13 | 14 | 15 | 17 | 13 | 17 | 19 |
| 20 | | | 1 | 12 | 12 | 17 | 18 | 13 | 6 | 11 | 15 | 14 | 17 | 11 | 16 | 18 |
| 21 | | | | 1 | 22 | 10 | 14 | 7 | 15 | 20 | 15 | 21 | 20 | 21 | 25 | 27 |
| 22 | | | | | 1 | 26 | 26 | 23 | 12 | 7 | 23 | 21 | 25 | 10 | 19 | 12 |
| 23 | | | | | | 1 | 6 | 6 | 20 | 24 | 10 | 17 | 14 | 19 | 21 | 25 |
| 24 | | | | | | | 1 | 10 | 21 | 23 | 6 | 14 | 9 | 18 | 17 | 24 |
| 25 | | | | | | | | 1 | 16 | 21 | 10 | 17 | 15 | 19 | 21 | 25 |
| 26 | | | | | | | | | 1 | 11 | 18 | 17 | 20 | 12 | 17 | 19 |
| 27 | | | | | | | | | | 1 | 19 | 18 | 22 | 7 | 16 | 12 |
| 28 | | | | | | | | | | | 1 | 11 | 8 | 15 | 15 | 21 |
| 29 | | | | | | | | | | | | 1 | 9 | 13 | 9 | 16 |
| 30 | | | | | | | | | | | | | 1 | 17 | 11 | 19 |
| 31 | | | | | | | | | | | | | | 1 | 11 | 8 |
| 32 | | | | | | | | | | | | | | | 1 | 13 |
| 33 | | | | | | | | | | | | | | | | 1 |

**Fig. 3.** The 33-zone travel time matrix.



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TABLE I

*Results of Conventional Set Covering Problem Applied to Austin Data*

| Maximum Allowable Travel Time | Objective Function Value | Multiple Coverage $\sum S_i$ | Nature of Decision Variables | Vehicles Located in Zones |
|---|---|---|---|---|
| 3 | 27 | 0 | Integer | 1, 2, 3, 4, 5, 9, 10, 11, 12, 13, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33 |
| 4 | 20 | 0 | Integer | 1, 2, 8, 13, 15, 16, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33 |
| 5 | 16 | 7 | Integer | 2, 10, 11, 17, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33 |
| 6 | 12 | 5 | Integer | 5, 10, 12, 16, 21, 22, 24, 27, 29, 30, 32, 33 |
| 7 | 8 | 4 | Integer | 11, 16, 22, 24, 29, 30, 32, 33 |
| 8 | 6 | 6 | Integer | 9, 20, 22, 25, 28, 33 |
| 9 | 6 | | Noninteger | |
| 10 | 3.75 | | Noninteger | |
| 11 | 3 | 24 | Integer | 19, 28, 31 |
| 12 | 2 | 11 | Integer | 15, 18 |
| 13 | 2 | 17 | Integer | 11, 16 |
| 14 | 2 | | Noninteger | |
| 15 | 1 | 0 | Integer | 7 |
| 16 | 1 | 0 | Integer | 6 |
| 17 | 1 | 0 | Integer | 4 |
| 18 | 1 | 0 | Integer | 4 |
| 19 | 1 | 0 | Integer | 2 |
| 20 | 1 | 0 | Integer | 2 |

Table I summarizes the output of the LP analog of the CSC formulation applied to the Austin data. The maximum allowable travel time, $T$, was varied from 3 to 20 minutes in 1-minute increments. In only 3 of the 18 cases was a noninteger solution obtained.

Table II presents the results of the LP analog of the HOSC problem when applied to the Austin data. The value of $W$ was 34 or $N + 1$. In all cases, the number of ambulances was the same as that found using the CSC formulation. However, the zones identified by the HOSC formulation differ from those found by the CSC model. This is due in part to Proposition 3, which ensures that dominated zones are excluded from the HOSC solution. For example, with a maximum allowable response time of 11 minutes, zone 2 dominates zone 19 as shown in Figure 3. Consequently, the HOSC solution includes zone 2 instead of zone 19. Also note that in two of the three cases in which the CSC problem terminated in a noninteger solution, the HOSC problem found an all integer solution.[2]

[2] When the same problems were run using the original 34 zones and a value of $W = 35$, the CSC problem terminated in a noninteger solution 6 out of 18 times. In 5 of those 6 cases, the HOSC formulation identified an all integer solution with the minimum number

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TABLE II

*Results of Hierarchical Objective Set Covering Problem Applied to Austin Data*

| Maximum Allowable Travel Time | Objective Function Value | Number of Vehicles Deployed | Multiple Coverage $\sum S_i^*$ | Nature of Decision Variables | Vehicles Located in Zones |
|---|---|---|---|---|---|
| 3 | 916 | 27 | 2 | Integer | 1, 2, 3, 4, 6, 7, 9, 10, 12, 13, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33 |
| 4 | 680 | 20 | 0 | Integer | 1, 2, 8, 13, 15, 16, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33 |
| 5 | 537 | 16 | 7 | Integer | 2, 10, 11, 17, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33 |
| 6 | 397 | 12 | 11 | Integer | 7, 10, 12, 16, 21, 22, 24, 27, 29, 30, 32, 33 |
| 7 | 258 | 8 | 14 | Integer | 4, 11, 16, 24, 27, 30, 32, 33 |
| 8 | 187 | 6 | 17 | Integer | 9, 20, 25, 27, 28, 31 |
| 9 | 165 | 6 | 39 | Integer | 8, 11, 16, 24, 27, 31 |
| 10 | 99 | 3.75 | | Noninteger | |
| 11 | 76 | 3 | 26 | Integer | 2, 28, 31 |
| 12 | 57 | 2 | 11 | Integer | 15, 18 |
| 13 | 43 | 2 | 25 | Integer | 3, 11 |
| 14 | 39 | 2 | 29 | Integer | 7, 11 |
| 15 | 34 | 1 | 0 | Integer | 7 |
| 16 | 34 | 1 | 0 | Integer | 6 |
| 17 | 34 | 1 | 0 | Integer | 4 |
| 18 | 34 | 1 | 0 | Integer | 4 |
| 19 | 34 | 1 | 0 | Integer | 2 |
| 20 | 34 | 1 | 0 | Integer | 2 |

The previously discussed constraint (4) was added to the CSC problem for the three cases in which the CSC formulation terminated in a noninteger solution. The results are shown in Table III. In only one of the three cases ($T = 14$) did addition of this constraint result in an integer solution. However, in that case, imposition of constraint (4) resulted in more than the minimum number of ambulances being deployed, as shown in Table II. This results from the fact that constraint (4) only applies if $Z_{1L}$ is itself noninteger; if it is integer, constraint (4) needs to be modified to read

$$\sum X_j \geq \langle Z_{1L} \rangle = Z_{1L}. \tag{8}$$

When constraint (8) was added to the two cases in which the CSC formulation terminated in a noninteger solution, one of the two new

of ambulances. In the sixth case, the HOSC problem terminated with a noninteger solution, which was identical to the CSC solution.

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TABLE III

*Results of Imposing Lower Bound on Number of Vehicles*

| Travel Time | CSC Objective Function Value | Constraint Value | Nature of Decision Variables |
|---|---|---|---|
| 9 | 6 | 6[a] | Integer |
| | | 7[b] | Noninteger |
| 10 | 3.75 | 4[b] | Noninteger |
| 14 | 2 | 2[a] | Noninteger |
| | | 3[b] | Integer |

[a] Indicates values found by imposing constraint (8).
[b] Indicates values suggested by following Toregas et al.[22] and imposing constraint (4).

solutions was integer. Since (8) is not a binding constraint in these cases, imposition of the constraint can result in an integer solution only if the order in which variables enter and leave the LP basis is altered by the constraint.

## 7. SUMMARY

IN THIS PAPER we formulated a hierarchical objective covering problem for locating EMS vehicles. The approach attempts to account explicitly for the importance of interdistrict responses. Several important properties of the formulation are derived including the fact that the solution to the multiobjective problem will not include dominated zones for a specified range of weights on the two objectives. The authors are currently studying hierarchical formulations of other location models, to determine whether or not similar properties may be derived.

Computational experience with the model is discussed for a 33-zone problem in Austin, Texas. Through an appropriate choice of the weights of the objectives in the objective function, the number of vehicles identified by the HOSC formulation always equaled the number found by the CSC formulation. However, the linear programming analog to the hierarchical objective set covering model terminated in an integer solution more often than did the LP version of the conventional set covering model.

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